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**6. AUTHOR(S)**

Levon Minnetyan

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**13. ABSTRACT (Maximum 200 words)**

A hybrid method based on standard nonlinear and linear numerical procedures was implemented for the complete simulation of structural response for vehicles taxing over an irregular surface. Time history integrations of the equations of motion were used to determine the nonlinear suspension forces on the basis of a small number of model coordinates. The time history results were used as inputs to a second stage linear analysis by which means the more detailed vehicle elastic response was computed. The current capabilities of the method were demonstrated by its application to a simple beam vehicle model and also by the transient simulation of a typical fighter aircraft, taxing over an irregular runway. The results were compared with test data and direct time history analysis. Numerical problems in the computer implementation of the hybrid method were examined and feasibilities for future modification and improvement of the solution procedure were indicated.

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# RESEARCH OBJECTIVES

The main objective of this research was to investigate alternative modeling techniques to simulate the transient dynamic response of partially nonlinear structures. The research proposal was based on the Summer research performed by the Principal Investigator during his 1981 SFRP research associateship at the Flight Dynamics Laboratory. A hybrid solution method for the efficient transient analysis of partially nonlinear structures was formulated during the SFRP research. The purpose of the present research was the implementation of the hybrid method and its comparison with standard time-history analysis and test data.

The first aim of the current research was to numerically investigate the hybrid solution method by applying it to a simple two-dimensional vehicle structure that was assumed to be taxiing over a irregular profile. Parametric studies with variations in the refinements of time domain discretization, modal decoupling, duration of transient simulation and modeling accuracy were aimed to evaluate the hybrid method as compared to time-history analysis.

The second task of this research was to use actual stiffness and mass data of a specific aircraft which had been tested. The objective of this last phase was to compare the method with test data and to assess its practical usefulness in simulating taxiing aircraft. Stiffness, mass, and modal data for a specific aircraft was provided by the Flight Dynamics Laboratory.

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## STATUS OF THE RESEARCH

A summary of the subject research is condensed in the two papers that are appended at the end of this report. Appendix I consists of a paper entitled "Dynamic Simulation of Structural Systems with Isolated Nonlinear Components" that will be published in the 53rd Shock and Vibration Bulletin in May 1983. The paper summarizes the formulation for transient analysis of a taxiing vehicle with a nonlinear suspension system. In numerical implementations, the vehicle is assumed to be a simple beam, idealized by a finite number of bending elements as shown in Fig. 1, Appendix I. Dynamic forces internal to the vehicle were simulated by a time-history analysis procedure using the TAXI program developed at FDL. Some modifications were made to TAXI to compute the internal forces in the beam vehicle. The same internal forces were also computed by the hybrid method as explained in the paper. Figs. 2 through 10 depict transient simulations by the two methods for different modal contents as implemented in this research. The major practical difficulty encountered in the numerical implementation was the requirement of large computer storage space in the hybrid method to store the modal time histories separately before final superposition. The needed computer storage space was proportional to the total simulated transient response period, as well as the number of modes considered and the refinement of the time stepsize. Thus, it was not possible to refine the hybrid method to the same stepsize as in time-history analysis. In the plots shown in the Appendix I paper, the time stepsize in the hybrid method is eight times coarser compared to the time stepsize in time-history analysis. The purpose of the paper is to compare the hybrid solutions with respect to one another to demonstrate the validity of the modal decoupling assumption.

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Comparison of hybrid simulations with time-history analyses can only be considered valid on a qualitative basis in matching the general characteristics of the two solutions. The computer memory constraint encountered in this research has indicated a new direction for the reorganization of the hybrid method to render a more practical numerical procedure. In future implementations of the hybrid method, the total transient response analysis time period will be subdivided into more convenient smaller periods to reduce the computer storage requirements and to make the hybrid method more interactive with time-history analysis. This avenue for future development of the hybrid method is indicated in the last paragraph of the Appendix I paper.

Appendix II consists of another paper entitled "Transient Response of Taxiing Aircraft" which is to be published in the Proceedings of the 24th SDM Conference in May 1983. This paper summarizes the work performed for the second task of the reported research. Modifications were made in the TAXI program to compute the wing root shear force and bending moment by summing up the applied loads and inertial forces on the wing of a tested aircraft. The nonlinear suspension forces are computed by TAXI and compared with test data to ascertain the reliability of the basic time-history suspension forces. The wing root shear force and bending moment for which test data is available are simulated using TAXI and the hybrid method similar to the Appendix I paper. TAXI simulations are generally closer to the experimental results. Nevertheless, for the majority of the peak dynamic loads, the test results fall between the TAXI and hybrid predictions. It is expected that TAXI and hybrid simulations will converge to the same solution with additional numerical and procedural refinements that are contemplated for the hybrid method in the future. Currently, a

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Master's thesis is in progress in which the computer storage problem is being addressed within the implementation of the present hybrid method. Once the numerical accuracy of the hybrid method is upgraded, other considered applications include the utilization of the procedure with a general purpose dynamic analysis program for the transient analysis of various structures with distinct nonlinearities such as space structures with nonlinear attitude control devices or building structures with non-linear fuse elements under seismic loading.

In summary, research results and conclusions can be stated within the following five items.

1. The hybrid analysis method has been implemented for the complete transient analysis of taxiing vehicles with nonlinear suspension systems.

2. The fundamental assumption of modal decoupling between the suspension response and the high frequency vibration modes has been verified.

3. Even though the implemented hybrid method was not to the same numerical accuracy as the reference time-history simulations, the hybrid solutions are stable and realistic in comparison with test data.

4. Test results are in general contained within the time-history and hybrid simulation results.

5. To overcome the computer memory constraints the hybrid method should be reorganized and applied separately to smaller subdivisions of the total transient analysis period.

# WRITTEN PUBLICATIONS

Minnetyan, L., Lyons, J.A., and Gerardi, T.G., "Dynamic Simulation of Structural Systems with Isolated Nonlinear Components," Shock and Vibration Bulletin, No. 53, Washington, D.C., May 1983.

Minnetyan, L., and Gerardi, T.G., "Transient Response of Taxiing Aircraft," Proceedings, 24th AIAA/ASCE/ASME/AHS joint Structures, Structural Dynamics and Materials (SDM) Conference, Lake Tahoe, Nevada, May 2-4, 1983.

# PROFESSIONAL PERSONNEL

Principal Investigator:    Levon Minnetyan, Assistant Professor
                           Clarkson College of Technology

# INTERACTIONS

Spoken Papers:

Minnetyan, L., Lyons, J.A., and Gerardi, T.G., "Dynamic Simulation of Structural Systems with Isolated Nonlinear Components," 53rd Shock and Vibration Symposium, Danvers, Massachusetts, October 26-28, 1982.

Minnetyan, L., and Gerardi, T.G., "Transient Response of Taxiing Aircraft," 24th Structures, Structural Dynamics and Materials (SDM) Conference, Lake Tahoe, Nevada, May 2-4, 1983.

APPENDIX I

Dynamic Simulation of Structural Systems with Isolated Nonlinear Components:

a paper to be published in the <u>Shock and Vibration Bulletin</u>, No. 53,
Washington, D.C., May 1983.

# DYNAMIC SIMULATION OF STRUCTURAL SYSTEMS

## WITH ISOLATED NONLINEAR COMPONENTS[*]

Levon Minnetyan
Clarkson College of Technology
Potsdam, New York

John A. Lyons
Niagara Mohawk Power Corporation
Syracuse, New York

Tony G. Gerardi
AFWAL/FIX
Wright-Patterson AFB, Ohio

Transient analysis of complex structural systems that contain both linear and nonlinear elements often presents a formidable computational problem. Nonlinear constitutive relationships in one or more of the structural elements necessitate a time-history integration of the equations of motion for the entire structure. Time-history analysis is usually carried out in terms of a limited number of generalized modal coordinates for the elastic substructure. However, a limited modal content is not sufficient to simulate the detailed structural response. To overcome this drawback a new hybrid method is formulated for the detailed dynamic analysis of complex structures. The new solution procedure incorporates a time-history analysis of the nonlinear response with a frequency domain analysis of the linear modes. The frequency domain analysis uses a larger number of modal coordinates to realistically simulate the details of structural response. The basic modal decoupling assumption of the hybrid method is studied by numerical application of the method to a simple elastic vehicle with nonlinear suspension properties, taxiing over an irregular profile.

## INTRODUCTION

A new hybrid method is developed for the complete simulation of the transient response of certain structural systems with specially designed nonlinear energy absorption and attitude control devices. Many complex systems contain both linear and nonlinear structural components. The types of structures that are particularly addressed in this paper are those with mostly linear characteristics. However, significant nonlinearities exist in some limited regions of the structure. Typically, nonlinear elements are connected to the remaining linear elastic substructure at a small number of nodes. Furthermore, the response of the nonlinear elements is primarily affected by the lowest or most fundamental frequencies and modes of vibration of the linear substructures.

The most immediate and relevant example of such a dynamic system is an aircraft taxiing over an irregular surface. Dynamic simulation of aircraft taxiing behavior requires a time-history integration of the equations of motion because of the typically nonlinear nature of the suspension strut properties. Nevertheless, the vehicle superstructure can be assumed to respond linearly for most aircraft. In theory, it is possible to simulate the total structural behavior by a direct dynamic analysis of a finite element model. However, the transient response

analysis of the full finite element model is prohibitively expensive. Also, past experience has shown that such analyses of complex structures can be numerically unreliable. The alternative to a direct finite element analysis is modal decomposition with substructuring that is being widely used at present for the simulation of aircraft taxiing response. Nonlinear terms are excluded from the dynamic equations by substructuring. For the linear substructures the vibration eigenvalue problem need be formulated only once at the beginning of the time-history solution. The modal super-position method is used for a change of basis from n nodal to p modal coordinates, $p \leq n$, prior to the time-history solution. The contributions of the nonlinear terms are treated as coordinate forces that are evaluated during the numerical integration of the equations of motion. If the objective of the time-history simulation is to predict the nonlinear suspension strut forces, the modal superposition series can be truncated after a few modes since the higher frequency vibrations do not affect the suspension response. Nevertheless, the dynamic response of some of the critical structural components may be significantly affected by high frequency modes. The necessity of including higher frequency vibration modes is especially relevant when force-related quantities such as internal loads are being estimated. Structural coordinate displacements can be expressed as the product of the mode shape matrix and the modal amplitude vector. On the other hand, the structure elastic forces can be written as the triple matrix product of the structure mass matrix, mode shape matrix, and a vector of modal dis-placements that are multiplied by the square of the corresponding modal frequencies. Since each modal contri-bution is multiplied by the square of its natural frequency, the higher frequency vibration modes are more important in simulating forces than displacements. If the transient behavior of such high frequency response quantities is to be predicted the original physical model must be sufficiently refined to yield meaning-ful vibration information on the high frequency modes which must be included in the modal superposition series. However, even if the high frequency vibration data is available to the required accuracy, the time-history analysis is not a useful technique to simulate the dynamic response of structural components that are affected by the high frequency modes. It is not practical to include the higher

frequency vibration modes with numerical reliability within the constraints of time-domain discretization. It would be necessary to decrease the time increment by several orders of magnitude to simu-late the higher frequency modes con-sistently. Such refinement of the time step with the addition of a greater number of modal coordinates makes the time-history analysis approach prohibi-tively expensive for design calcu-lations. A reliable and efficient method is needed to simulate total structural response for comparison with critical stress limits or to establish relevant design criteria.

To accomplish this task, a hybrid analytical method is formulated: aimed at defining an optimal solution path that will reliably predict dynamic response. The method incorporates a time-history analysis for the nonlinear response with a frequency domain analysis of the linear modes. First the time-history analysis including the non-linear components and a small number of linear modes is conducted. Partial decoupling of the nonlinearities from the rest of the structure constitutes the second step. The remaining linear dynamic subsystem is analyzed through the frequency domain under external forces and interactions from the non-linear components.

The numerical objective of this paper is to validate the fundamental assumption made in the new hybrid formu-lation. The numerical results presented are chosen to corroborate partial de-coupling as a realistic procedure. Not-withstanding with the fact that the new method is envisioned as a means for efficient dynamic simulation for complex structures, the first numerical appli-cation is made to a rather simple two-dimensional beam vehicle that is taxiing over an irregular profile. The choice of a simple beam to represent an elastic vehicle is to have complete assurance and control on the finite element model during this first stage validation of the hybrid method. The results show a reasonable justification of the modal decoupling assumption and indicate possible directions of research for the improvement and development of the method.

Although the hybrid simulation method is examined primarily with reference to the taxiing vehicle problem, it should be noted that other examples of physical systems can also be found in the same dynamic category. In general any combination of linear elastic substructures coupled together by nonlinear energy absorption and

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dissipation devices can be analyzed with the developed method. Specific examples include earthquake resistant buildings incorporating nonlinear "Safe Fuse" elements and space structures that are composed of linear elastic substructures assembled together by means of nonlinear couplers that absorb vibration energy and are used for attitude control adjustments.

## FORMULATION OF THE NONLINEAR PROBLEM

In general, the structure nodal coordinate dynamic equilibrium equations can be written as [1]

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{P\} \quad (1)$$

where

[m] = structure nodal coordinate mass matrix

{u} = list of nodal coordinate displacements

[c] = structure damping matrix

[k] = structure piecewise linear tangent stiffness matrix (includes both constant and variable stiffness coefficients)

{P} = list of external loads

Eq. (1) is not convenient for representation in terms of an orthogonal modal basis. When the structure stiffness matrix is reassembled at each solution step, it becomes also necessary to redefine and reanalyze the vibration eigenvalue problem.

The transformation to modal coordinates becomes practical only if the nonlinearities are accounted for in terms of additional effective loads. To achieve this, the total stiffness matrix [k] in Eq. (1) is separated into its linear and nonlinear components

$$[k] = [k^o] + [k^{NL}] \quad (2)$$

where

$[k^o]$ = constant linear elastic stiffness matrix

$[k^{NL}]$ = nonlinear stiffnesses which depend on the state of deformation

Substituting Eq. (2) into Eq. (1)

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k^o]\{u\} + [k^{NL}]\{u\} = \{P\} \quad (3)$$

The nonlinear effects can be expressed as additional coordinate loads:

$$\{P_{NL}\} = [k^{NL}]\{u\} \quad (4)$$

the equations of dynamic equilibrium are written as

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k^o]\{u\} = \{P\} - \{P_{NL}\} \quad (5)$$

The effective $\{P_{NL}\}$ loads are determined at each time step from nonlinear component properties.

To change the solution basis from n nodal to p modal coordinates, $p \leq n$, an orthogonal transformation is written

$$\{u\} = [\phi]\{\eta\} \quad (6)$$
$$nx1 \quad nxp \ px1$$

where $\eta$ are the generalized modal coordinates and $[\phi]$ is the mode shape matrix whose columns are the orthogonal eigenvectors $\{\phi\}$ of the vibration problem defined by

$$[k^o]\{\phi\} = \omega^2[m]\{\phi\} \quad (7)$$

where $\omega$ is the natural frequency corresponding to $\{\phi\}$. The vibration problem need be formulated only once at the beginning of time-history analysis due to the exclusion of the variable components from the stiffness matrix $[k^o]$. The equations of motion can be written in modal coordinates

$$[M]\{\ddot{\eta}\} + [C]\{\dot{\eta}\} + [K]\{\eta\} = [\phi]^T\{P - P_{NL}\} \quad (8)$$
$$pxp\,px1 \quad pxp\,px1 \quad pxp\,px1 \quad pxn \ nx1$$

where

[M] = generalized mass matrix;
$[M] = [\phi]^T[m][\phi]$

[C] = modal damping matrix;
$C_{jj} = 2\zeta_j\omega_j M_{jj}, \ C_{ij} = 0$

[K] = modal stiffness matrix;
$K_{jj} = \omega_j^2 M_{jj}, \ K_{ij} = 0$

$\zeta_j$ = jth modal damping ratio

The time-history integration of the equations of motion can now be accomplished more efficiently in terms of the modal coordinates. It should be noted that although time-history integration is performed using the modal coordinates, some nodal displacements must be calculated at each time step to evaluate the effective load vector $P_{NL}$ in Eq. (8). Generally this does not pose a problem since the effective loads usually act only at a small number of nodal coordinates.

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It is relevant to take note of the
modeling requirements for the free
vibration problem of a linear sub-
structure within the context of the
orthogonal formulation given by Eq. (8).
In Eq. (8), each linear substructure is
considered separately from the
surrounding nonlinear components. The
interactive forces between a sub-
structure and its surroundings are
considered as external loads at the sub-
structure boundaries.

The linear elastic substructure is
represented by a finite element model
that consists of a number of nodes
connected by idealized discrete ele-
ments. The required refinement of the
finite element model depends upon
structural geometry, boundary con-
ditions and applied loading. The
vibration eigenvalue problem for the
linear substructure is defined by Eq.
(7). In order that the modal equations,
Eq. (8), be entirely orthogonal includ-
ing the rigid body modes, the stiffness
$[k^o]$ and mass $[m]$ matrices in Eq. (7)
are defined for the unconstrained
structure. In Eq. (7) the mass matrix
is positive definite, the stiffness
matrix is semi-definite. Another
requirement for the orthogonality of the
elastic vibration modes to the rigid
body modes is the definition of the mode
shape vectors $\{\phi\}$ in Eq. (7) relative to
the dynamic center of mass of the
flexible substructure. The dynamic
center of mass can be defined as the
instantaneous center of structural mass
during dynamic response. The behavior
of the dynamic center of mass is
described by the rigid body modes of
motion. Thus, the dynamic center of
mass will remain stationary during free
vibrations of an unconstrained
structure [2].

In general, any structure
coordinate displacement can be expressed
as the sum of rigid body displacement
contributions plus the effect of elastic
structural deformations. The elastic
deformations can be expressed in terms of
the amplitudes of flexible vibration
modes. If we write the modal super-
position equations for all structure
coordinates in matrix form then

$$\{u\} = [\phi_R] \{u_R\} + [\phi_F] \{q\} \qquad (9)$$

where

$\{u_R\}$ = vector of rigid body
displacements

$[\phi_R]$ = vector or rigid body modal
influence coefficients.
Each column of $[\phi_R]$ lists
the displacements at

structure coordinates due
to a unit displacement of
the corresponding rigid
body coordinate.

$[\phi_F]$ = mode shape matrix of the
flexible modes. Each
column of $[\phi_F]$ represents
a mode shape vector.

$\{q\}$ = flexible mode amplitudes

The superposition equations, Eq.
(9) may be combined into a single
matrix of rigid body plus flexible modal
influence coefficients

$$\{u\} = [\phi_R \ \phi_F] \left\{ \frac{u_R}{q} \right\} \qquad (10)$$
$$n \times 1 \qquad n \times p \qquad p \times 1$$

defining new symbols for the combined
matrices

$$\{u\} = [\phi] \{\eta\} \qquad (11)$$

which is the same as Eq. (6). It
should be noted that the rigid body
modes must have the specific scales to
render the generalized mass equal to
the physical mass or inertia of the
corresponding rigid body mode.
Accordingly, in Eq. (9) if $u_i$ is in the
same direction as the jth rigid body
mode $u_{Rj}$, then $\phi_{Rij}$ is the moment arm
of $u_i$ from the jth rigid body rotation
axis [2].

COMPUTATION OF STRUCTURAL RESPONSE

The first step is the computation
of time-history response of nonlinear
components. A time-history integration
of the modal equations of motion, Eq.
(8), is performed including a small
number of modes that are sufficient to
represent the flexible deformations of
the elastic structure for the purpose
of estimating the behavior of the non-
linear components. Although the mass,
damping and stiffness matrices are
diagonal in Eq. (8), the time-history
integration of the equations must
progress simultaneously since the
equations are coupled with $P_{NL}$ nonlinear
terms on the right hand side.

After the time-history
determination of the nonlinear forces
the total structural response is
evaluated through the frequency domain.
The basic requirements for a frequency
domain analysis are that the nonlinear
interaction forces are known and the
linear systems are represented by ortho-
gonal generalized coordinates. At this
stage a much greater number of modes can
be included to represent the linear

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substructure in detail for the determination of its internal response due to the numerical stability and efficiency of frequency domain analysis.

The time-history dynamic forces, including the nonlinear interaction forces, acting on the linear substructure are first converted to the frequency domain by Discrete Fourier Transformation (DFT). The DFT coefficients are defined as [1]

$$C_{n+1}(\bar{\omega}_n) = \Delta t \sum_{t=0}^{N-1} F(t) e^{-2\pi i n t/N};$$

$$n = 0, \ldots, N-1 \quad (12)$$

where

i $= \sqrt{-1}$

$\Delta t$ $= T/N$

T $=$ total time period considered (includes an attached period of $F(t) = 0$ to take into account the periodic nature of DFT)

N $=$ number of discrete time intervals in T

$\bar{\omega}_n$ $=$ forcing frequency

$C(\bar{\omega}_n)$ $=$ coefficients defining the discretized harmonic amplitude function

The Complex-Frequency-Response-Function (CFRF), $H_j(\bar{\omega}_n)$, for each jth generalized structural mode under the forcing frequency $\bar{\omega}_n$ is defined as [1]

$$H_j(\bar{\omega}_n) = \frac{1}{-\bar{\omega}^2 M_{jj} + i\bar{\omega}_n C_{jj} + K_{jj}} \quad (13)$$

where

i $= \sqrt{-1}$

$M_{jj}, C_{jj}, K_{jj}$ $=$ jth generalized modal mass, damping, and stiffness, respectively

For the rigid body modes $C_{jj} = K_{jj} = 0$, and

$$H_j(\bar{\omega}_n) = \frac{1}{-\bar{\omega}_n^2 M_{jj}} \quad (14)$$

It can be shown that the total response of a system to any forcing input can be written by means of Inverse Fourier Transformations (IFT). The displacements of the jth modal coordinate are

given by [1]

$$\eta_j(t) = \frac{\Delta\omega}{2\pi} \sum_{n=0}^{N-1} H_j(\bar{\omega}_n) C_j(\bar{\omega}_n) e^{i\bar{\omega}_n t} \quad (15)$$

or since $\bar{\omega}_n = n\Delta\omega$ and $\Delta\omega = \bar{\omega}_n/N$; (16)

$$\eta_j(t) = \frac{\Delta\omega}{2\pi} \sum_{n=0}^{N-1} H_j(n+1) C_j(n+1) e^{2\pi i n t/N} \quad (17)$$

Both the DFT harmonic amplitude coefficients of the generalized forces, Eq. (12), and the IFT to solve for generalized displacements, Eq. (17), can be rapidly generated by modern Fast Fourier Transform (FFT) algorithms.

Modal accelerations can be evaluated from the second time derivative of Eq. (17)

$$\ddot{\eta}_j(t) = \frac{\Delta\omega}{2\pi} \sum_{n=0}^{N-1} H_j(n+1) C_j(n+1)$$

$$\left(\frac{-4\pi^2 n^2}{N^2}\right) e^{2\pi i n t/N} \quad (18)$$

The structure nodal coordinate displacements can be obtained from the modal superposition equations

$$\{u(t)\} = [\phi] \{\eta(t)\} \quad (19)$$
$$n\times1 \quad n\times p \quad p\times1$$

Other response parameters such as stresses or loads developed in various structural components can be evaluated directly from the displacements. For example, the elastic forces {f} which resist the deformation of the structure are given directly by the displacements and the structure stiffness coefficients.

$$\{f(t)\} = [k^o]\{u(t)\} = [k^o][\phi]\{\eta(t)\} \quad (20)$$

An alternative expression for the elastic forces can be written in terms of the structure mass matrix and natural frequencies. Expanding Eq. (20) in terms of the modal contributions

$$\{f(t)\} = [k^o]\{\phi_1\}\eta_1(t) + [k^o]\{\phi_2\}\eta_2(t)$$

$$+ \ldots + [k^o]\{\phi_p\}\eta_p(t) \quad (21)$$

Substituting Eq. (7) in each term of Eq. (21)

$$\{f(t)\} = \omega_1^2[m]\{\phi_1\}\eta_1(t) + \omega_2^2[m]\{\phi_2\}\eta_2(t)$$

$$+ \ldots + \omega_p^2[m]\{\phi_p\}\eta_p(t) \quad (22)$$

Combining back into matrix form

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$$\{f(t)\} = [m][\phi]\{\omega_j^2 \hat{\eta}_j(t)\} \quad (23)$$

where $\{\omega_j^2 \eta_j(t)\}$ represents a vector of modal amplitudes, each multiplied by the square of its modal frequency.

In Eq. (20), if $[k^o]$ is the substructure stiffness matrix and $\{u(t)\}$ is the list of substructure coordinate displacements then $f(t)$ are the total elastic forces at the substructure nodal coordinates. To obtain the internal stresses at particular locations in the substructure, the elemental stiffness matrices and the corresponding element vertex displacements are used.

$$\{f'(t)\} = [k']\{u'(t)\} \quad (24)$$

where prime indicates the quantities that are defined with respect to elemental vertex coordinates.

APPLICATION TO TAXIING VEHICLE

The basic premise of the formulated hybrid method; that the response of the nonlinear components is influenced primarily by the most fundamental vibration modes and frequencies must be verified by numerical application of the method to a model problem. A simple taxiing vehicle model is used to compare the new method with time-history analysis and to validate the practicality of partial modal deccupling. The physical model consists of a two-dimensional simple beam vehicle taxiing over an irregular profile. The choice of a taxiing vehicle example at this first stage of numerical verification is due to the availability of the FDL-TAXI program that is used for time-history analysis. The FDL-TAXI program is a relatively simple program, yet it has been validated by comparison to extensive testing. Details of the TAXI program are well documented in reference [3].

The choice of a simple beam to represent an elastic vehicle is to have complete assurance and control on the finite element model. The vehicle model used is depicted in Fig. 1. The vehicle is represented by twelve beam elements. Individual beam elements are taken to be 100 in. long and weigh 4608 lbs each with a bending rigidity of $29\times10^8$ lbs-in$^2$. These elemental properties are selected to render the overall weight of the vehicle compatible with the suspension properties that are designed for a vehicle of about 55,000 lbs. The suspension system consists of nonlinear, oleo-pneumatic energy absorption devices. Typical nonlinear landing gear load-deflection relation-

ships are used to represent the suspension gear properties. Each suspension strut force is represented as the sum of pneumatic spring force, hydraulic damping force and strut friction force. The procedures in the existing FDL-TAXI program are used to model the suspension system similar to that of a typical fighter aircraft [4].

Prior to the transient analysis it is necessary to define the vibration problem and to determine the unconstrained vibration modes and the natural frequencies of the linear vehicle substructure. The vibration problem for the unconstrained vehicle is solved by an eigensolution algorithm based on the generalized Jacobi method with eigenvalue shifts [1]. The solution of the eigenproblem supplies the vibration mode shape, natural frequency and generalized mass data which is needed both in time-history and frequency domain analyses. Once the free vibration properties are determined, nonlinear time-history simulations are made using various dimensions for the modal basis as defined by Eq. (8). Two percent modal damping is assumed for each flexible mode. For the simple vehicle model with 12 elements and 26 geometric coordinates a maximum of 14 modes are considered including 2 rigid body modes and 12 flexible modes. In theory it would be possible to include the entire set of 26 modes. However, it has been found that the vibration problem solution for the higher modes is susceptible to numerical inaccuracies. It has been observed that only the first half of the vibration modal data is numerically reliable [5]. Accordingly, time-history simulations are made using 4, 8 and 14 orthogonal modes. The vehicle is assumed to travel at a constant velocity of 44 ft/sec. The total runway length traveled is 230 ft. with a standard 78 ft. AM-2 mat beginning at 106 ft. The AM-2 mat is 1.5 in. high and includes 4 ft. linear ramps at both ends. The same time discretization interval of 0.00025 seconds is used in all time-history analyses. This is the maximum discretization interval which can be used to secure numerical convergence with time-history integration when the maximum number of 14 modes are included. With the 14-mode time-history analysis the vehicle structure geometric coordinate response is also computed for later comparison with the hybrid results. Internal loads are chosen as controlling test cases because of the inherent difficulty of predicting them on the basis of a small number of modes. Specifically, the internal bending moment at coordinate 4 and the shear

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forces at coordinates 5 and 9, as referenced by Fig. 1, are presented in comparative results in Fig. 2 through Fig. 10. In these figures the time increment is 0.002 seconds which is the same as the discretization interval in frequency domain analysis and the horizontal axis spans 5.2 seconds. The broken lines represent the time-history simulations and the solid lines represent the hybrid simulations using the time-history nonlinear suspension forces combined with frequency domain analysis. All time-history curves (broken lines) are computed on the basis of 14 orthogonal generalized modes used in the TAXI program. All hybrid curves also use 14 modes in the frequency domain analysis. However, in the hybrid method the time-history non-linear forces that are converted to the frequency domain are computed on the basis of various number of modes. The different hybrid solutions are distinguished from one another on the basis of the modal content represented in their nonlinear force time-history inputs.

In Figs. 2, 3, and 4 comparisons between the hybrid method and time-history integration for the dynamic response of the internal bending moment at coordinate 4 is presented. In Fig. 2 both the hybrid method and the time-history integration are based on 14 modes. The initial phase of the response, which is seen as the relatively flat portion of the plot, corresponds to the perfectly flat portion of the runway. Theoretically, the dynamic response in this region should not be different from zero. However, both the hybrid and time-history integration schemes as implemented in this first study are vulnerable to small numerical perturbations, seen here as a deviation from a zero response. The previously described AM-2 mat is encountered by the taxiing vehicle at time increment 1205. The dynamic response of interest appears after the mat is encountered. The hybrid method simulation (solid line) is seen to follow the character of the response as calculated by complete time-history integration quite closely. There are differences, however, in the peak values as predicted by the two methods. It is not known at this time which solution gives a better estimate of the peak values. However, it is expected that the time-history solution will overshoot the peaks because of numerical constraints and the hybrid solution may flatten them out. It is expected that improvement in the numerical algorithms used to obtain both the time-history and frequency domain analysis will eliminate much of the discrepancy seen in these comparisons.

Fig. 3 shows the dynamic response of the same internal bending moment at coordinate 4 as in Fig. 2. However, in the hybrid method the number of modes included in the determination of the nonlinear strut forces has been reduced to 8 from the reference value of 14. The time-history integration plot remains the same as in Fig. 2, containing 14 modes. Comparison of the two curves in Fig. 3 again shows that the dynamic response as predicted by the two methods is characteristically in good agreement. Comparing Figs. 2 and 3, it is seen that the reduction of the number of modes in the hybrid analysis has had very little effect on the response simulation. A preliminary conclusion may thus be drawn that apparently the nonlinear suspension response can be represented with negligible error on the basis of 8 modes as in Fig. 3, compared to a basis of 14 modes as in Fig. 2. Thus, the comparison of the solid line plots of the hybrid method from Figs. 2 and 3 demonstrate the validity of partial modal decoupling assumption for this example.

Fig. 4 again shows the bending moment response at coordinate 4. The time-history integration with 14 modes and the hybrid solution with only 4 modes including two rigid body and two flexible modes are plotted for comparison. It is immediately obvious that the hybrid simulation in Fig. 4 differs greatly in character and magnitude from the time-history solution. It is concluded that 2 flexible modes are simply too few to adequately represent the vehicle elastic behavior in the computation of the dynamic response of the nonlinear suspension struts.

Figs. 5, 6, and 7 are analogous to Figs. 2, 3, and 4, respectively in comparing the hybrid simulations with time-history analysis. The dynamic response of the internal shear force at vehicle coordinate 5, as referenced by Fig. 1, is plotted in Figs. 5, 6, and 7. Fig. 5 shows the hybrid solution on the basis of 14 modes. Figs. 6 and 7 show the hybrid solution on the basis of 8 and 4 modes, respectively, included in the determination of the nonlinear suspension forces. The numerical perturbations discussed in Figs. 2, 3, and 4 are similarly present in Figs. 5, 6, and 7. Nevertheless, comparing Figs. 5 and 6 it is again observed that the 8 mode simulation give as good an approximation as does the 14 mode simulation. Also, the 4 mode simulation is again inadequate as can be seen from Fig. 7.

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The dynamic response of the internal shear force at coordinate 9 is shown in Figs. 8, 9, and 10. Coordinate 9 is of special interest because it is the attachment point for the main suspension gear strut. Fig. 8 shows the hybrid solution with 14 modes. Figs. 9 and 10 show the hybrid solution with 8 and 4 modes, respectively. From these three figures it is seen that the numerical perturbations in the initial smooth profile differ significantly from zero. However, comparison of this response as predicted by the hybrid method in each of the Figs. 8, 9 and 10 shows that these non zero perturbations are most likely contained in only the lowest modes, possibly the rigid body modes. Even though the response differs from the expected zero response in the initial phase, the character of the hybrid response curve still follows the general character of time-history simulation plot. As before, there is virtually no difference between the hybrid simulations containing 14 and 8 modes, as depicted in Figs. 8 and 9, respectively. Again, however, the 4 mode hybrid response of Fig. 10 is inadequate for predicting the dynamic behavior.

CONCLUSION

The basic premise of the hybrid method, that the nonlinear response in a structure with discrete nonlinear components is influence primarily by the low frequency vibration modes of the linear substructure, has been verified in a preliminary way. In the case of the vehicle model which has been the subject of numerical work in this paper, the nonlinear component response was successfully modeled by including only six flexible modes or one quarter of the total number of flexible modes of the linear substructure. It is thus concluded that the postulated partial modal decoupling of the nonlinear components from the linear structure actually occurs, making the hybrid solution method a viable approach for the analysis of structures with distinct nonlinearities. In practical cases where the total number of modes which must be included to adequately represent the dynamic response of the linear structures is quite large, it is expected that the hybrid method will greatly reduce the computational effort required in dynamic simulations.

The promise of the hybrid method has been demonstrated, however, the results reported in this paper must be regarded as preliminary. Additional research is needed before the hybrid method can be developed into a reliable and efficient analysis tool. Specifically, more information must be obtained on numerical requirements such as the maximum time-step size constraint to achieve reasonable accuracy and the modal content needed for a realistic solution for large and complex systems. Studies must be done to better understand the nature of partial decoupling of the nonlinear component response from the higher frequency vibration modes of the linear substructure. In this regard the extent of decoupling which could be expected to occur in actual complex structures must be determined in a realistic way. In the study presented in this paper the model structure was a simple 26 degree-of-freedom system and the nonlinear component response was adequately represented by one quarter of the flexible modes. However, in larger and more complex systems, the required number of modes for simulation of the nonlinear forces is not expected to increase in proportion to the degree of complexity of the problem. The nonlinear response is expected to be contained within the lowest 12 modes even for very large systems. This assumption must be verified through extensive testing of the new hybrid method.

Finally, as a result of the particular numerical insight gained in this paper, it seems appropriate to contemplate the reorganization of the hybrid method to act more interactively with time-history integration. Namely, a piecewise linear hybrid iterative method is envisioned in future work in which nonlinear time-history integration and linear structural analysis proceed in parallel with mutual checking and corrections.

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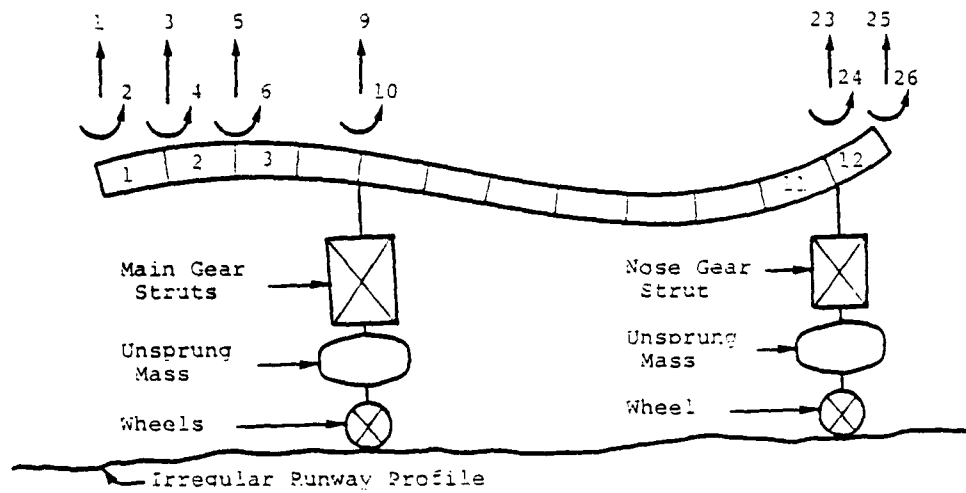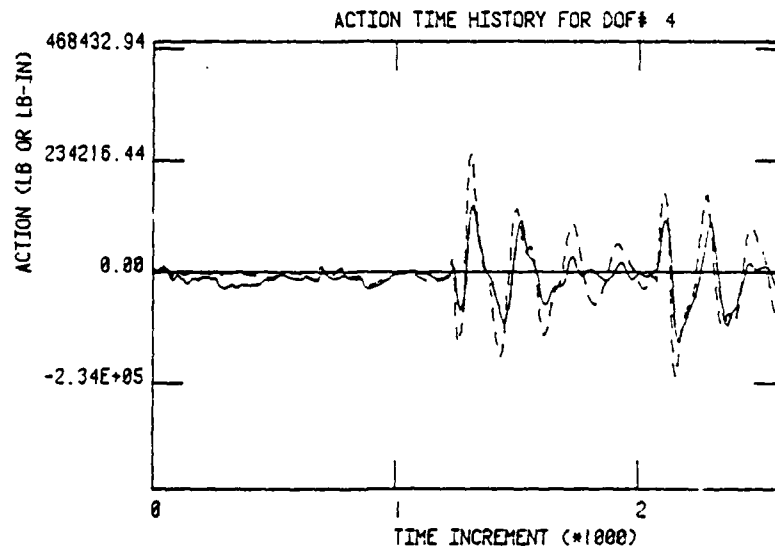    Hill Book Company, 1968.

Minnetyan



Fig. 1.  Simple Beam Vehicle Model



Fig. 2.  Bending Moment at Coordinate 4
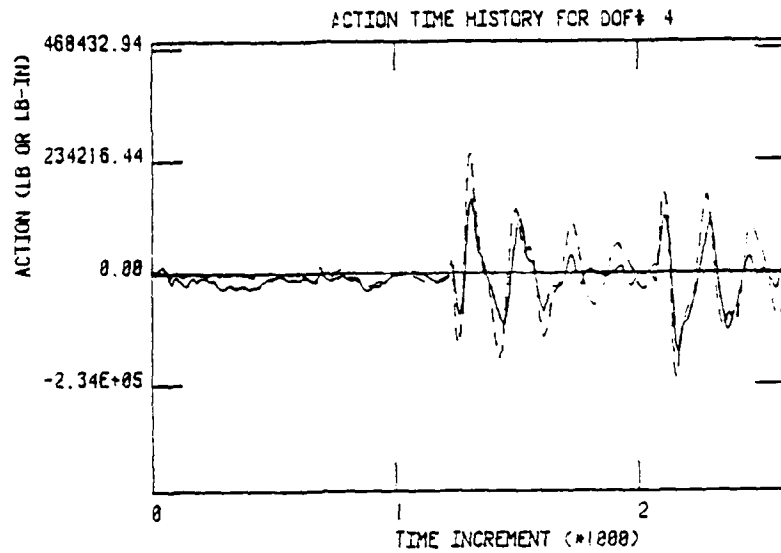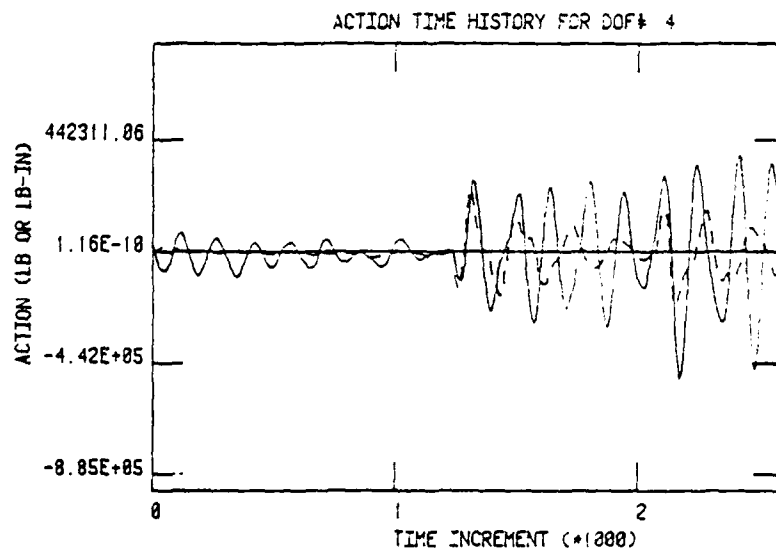Solid: Hybrid Simulation with 14 Modes
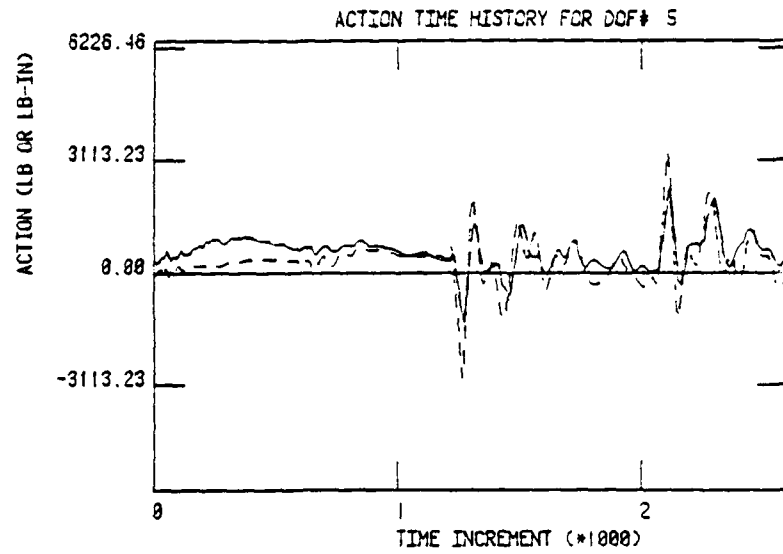Dashed: Time History Simulation with
14 Modes

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ACTION TIME HISTORY FOR DOF# 4



Fig 3.   Bending Moment at Coordinate 4
         Solid: Hybrid Simulation with 3 Modes
         Dashed: Time History Simulation with
              14 Modes

ACTION TIME HISTORY FOR DOF# 4



Fig. 4.   Bending Moment at Coordinate 4
          Solid: Hybrid Simulation with 4 Modes
          Dashed: Time History Simulation with
               14 Modes

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ACTION TIME HISTORY FOR DOF# 5



Fig. 5.   Shear Force at Coordinate 5
          Solid: Hybrid Simulation with 14 Modes
          Dashed: Time History Simulation with
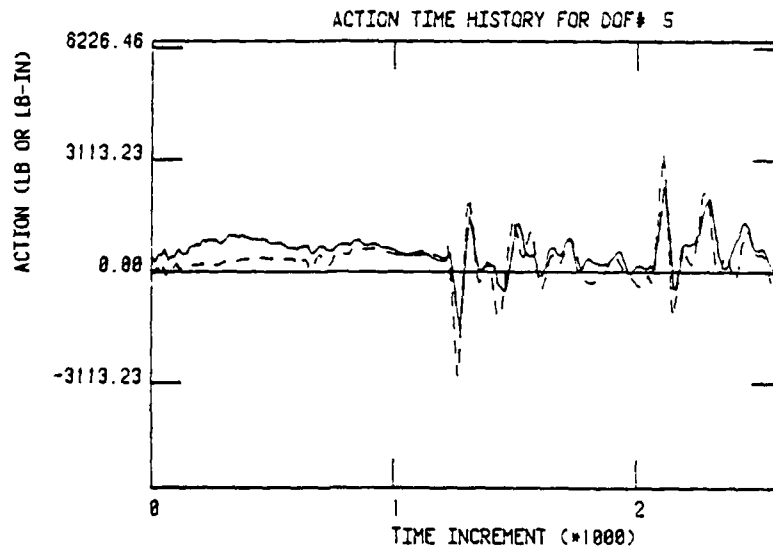                14 Modes

ACTION TIME HISTORY FOR DOF# 5



Fig. 6.   Shear Force at Coordinate 5
          Solid: Hybrid Simulation with 8 Modes
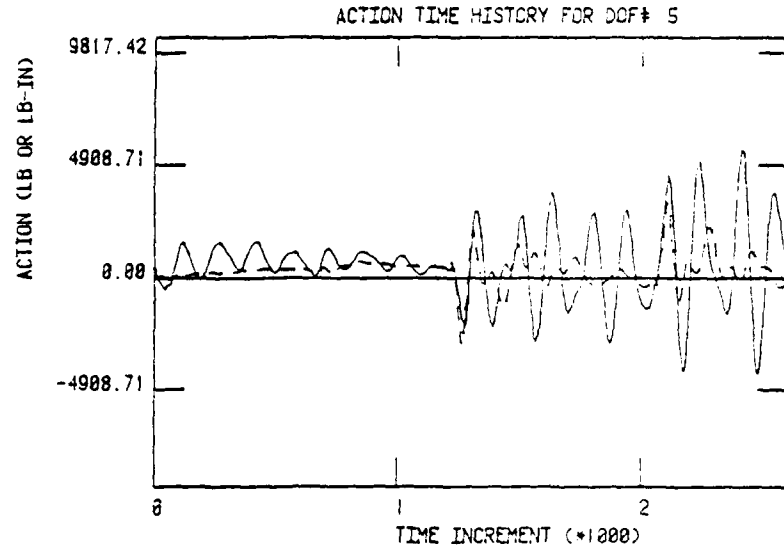          Dashed: Time History Simulation with
                14 Modes

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ACTION TIME HISTORY FOR DOF# 5

Fig. 7.   Shear Force at Coordinate 5
          Solid: Hybrid Simulation with 4 Modes
          Dashed: Time History Simulation with
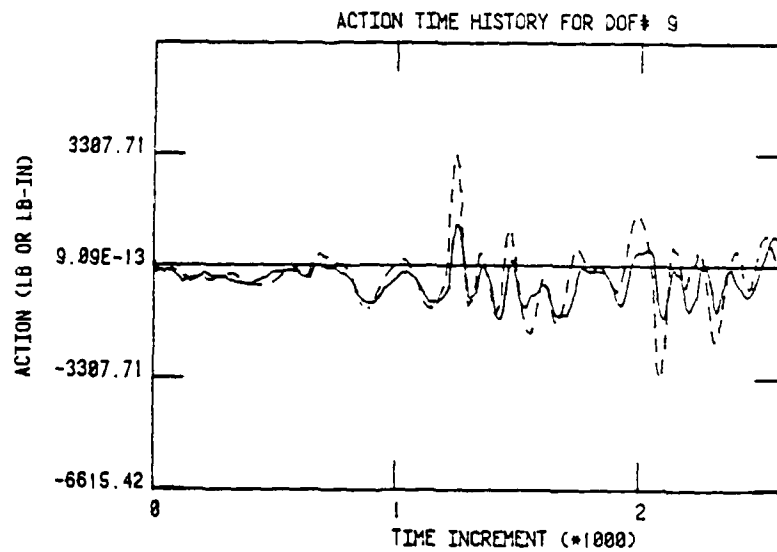                  14 Modes

ACTION TIME HISTORY FOR DOF# 9

Fig. 8.   Shear Force at Coordinate 9
          Solid: Hybrid Simulation with 14 Modes
          Dashed: Time History Simulation with
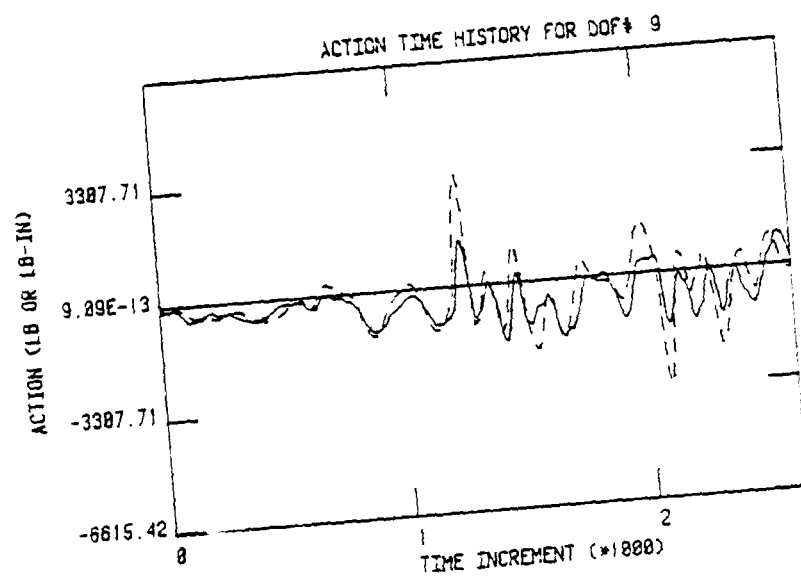                  14 Modes

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ACTION TIME HISTORY FOR DOF# 9



Fig. 9.  Shear Force at Coordinate 9
Solid: Hybrid Simulation with 3 Modes
Dashed: Time History Simulation with
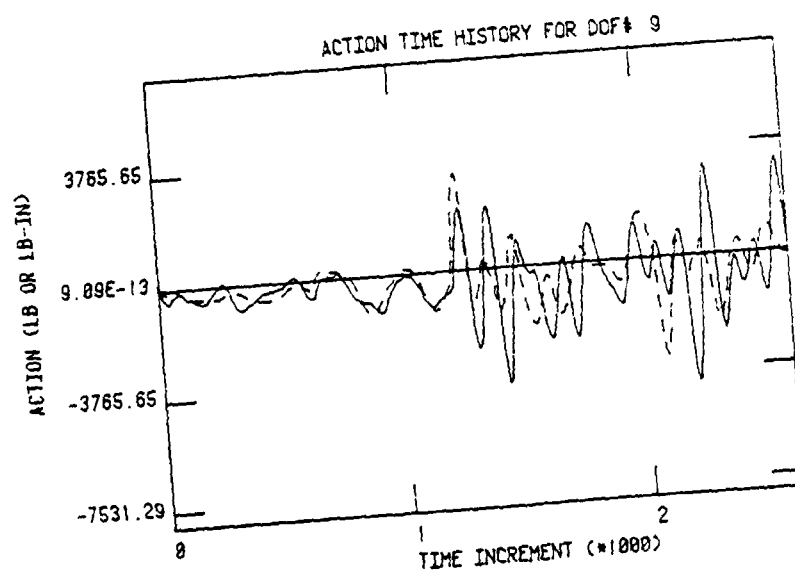14 Modes

ACTION TIME HISTORY FOR DOF# 9



Fig. 10.  Shear Force at Coordinate 9
Solid: Hybrid Simulation with 4 Modes
Dashed: Time History Simulation with
14 Modes

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APPENDIX II

Transient Response of Taxiing Aircraft:

A paper to be published in the Proceedings of the 24th AIAA/ASCE/ASME/AHS joint Structures, Structural Dynamics and Materials (SDM) Conference, Lake Tahoe, Nevada, May 2-4, 1983.

TRANSIENT RESPONSE OF TAXIING AIRCRAFT*

Levon Minnetyan**
Clarkson College of Technology
Potsdam, New York

Tony G. Gerardi[+]
Air Force Wright Aeronautical Laboratories
Wright-Patterson AFB, Ohio

## Abstract

A hybrid method based on standard
nonlinear and linear numerical procedures
is developed for the complete simulation
of structural response for aircraft taxi-
ing over an irregular runway. The method
uses time history integrations of the
equations of motion to determine the non-
linear suspension forces on the basis of a
small number of modal coordinates. The
time-history results are used as inputs to
a second stage linear analysis by which
means the more detailed structural re-
sponse is computed. The current capabili-
ties of the method are demonstrated by its
application to the simulation of a typical
fighter aircraft taxiing over an irregular
runway. The results are compared with test
data and direct time-history analysis.

## Introduction

Dynamic simulation of aircraft taxiing
behavior requires a time-history inte-
gration of the equations of motion because
of the typically nonlinear nature of the
suspension strut properties. However, the
vehicle superstructure can be assumed to
respond linearly for most aircraft. To
account for the vehicle flexibility during
taxiing, the fundamental elastic vibration
modes should be included in the model. If
the objective is to simulate only the sus-
pension response, the modal superposition
series can be truncated after a few modes
since the higher frequency vibrations do
not affect the suspension behavior. For
typical aircraft, the first five to ten
flexible modes are usually sufficient to
accurately predict the dynamic loads in the
suspension struts. Nonetheless, a signifi-
cantly larger number of flexible modes need

be included in the model if the behaviors
of other critical vehicle components are
to be simulated. The necessity of in-
cluding higher frequency vibration modes
is especially relevant if force-related
quantities such as inertial loads are
being estimated. However, the time-
history integration technique is not
practical to simulate the response of all
critical aircraft components; especially
those response quantities that are af-
fected by the high frequency modes. It is
not practical to include the higher
frequency vibration modes with numerical
reliability within the constraints of time-
domain discretization. It would be neces-
sary to decrease the time increment by
several orders of magnitude to simulate
the higher frequency modes consistently.
Such refinement of the time step with the
addition of a greater number of modal co-
ordinates makes the time-history analysis
approach prohibitively expensive for
design calculations. A simple, efficient,
and reliable method is needed to simulate
aircraft total structural response for
comparison with critical design limits to
establish relevant runway repair criteria.

To accomplish this task, a hybrid
analytical method is formulated: aimed at
defining an optimal solution path that
will reliably predict dynamic response.
The method incorporates a time-history
analysis for the nonlinear response with a
frequency domain analysis of the linear
modes. First the time-history analysis
including the nonlinear components and a
small number of linear modes is conducted.
Partial decoupling of the nonlinearities
from the rest of the structure constitutes
the second step. The remaining linear
dynamic subsystem is analyzed through the
frequency domain under external forces and
interactions from the nonlinear components.

The basic assumptions which permit a
frequency domain analysis are that the
nonlinear suspension interaction forces
are known and the linear vehicle super-
structure is represented by orthogonal
generalized coordinates. A much greater
number of modal coordinates can be ef-
ficiently included in the frequency domain
analysis as compared to time-history analy-
sis to represent the linear vehicle
structure in detail. The time-history
dynamic forces acting on the vehicle

structure are first converted to modal co-ordinates. A standard fast Fourier transform (FFT) algorithm is used to transform modal force histories to the frequency domain. The resulting discretized harmonic amplitude coefficients are combined with the complex frequency response functions of the orthogonal vibration modes. The frequency response is transformed back to the time domain by an inverse FFT. The results are the time histories of the generalized modal displacements. Superposition of the modal displacements yields the actual structure coordinate response.

The hybrid method was first evaluated by its application to a simple two-dimensional vehicle that is taxiing over an irregular profile. Analyses were carried out on the basis of a various number of generalized vibration modes to validate the modal decoupling assumption. In the present paper the application of the hybrid method to the simulation of taxiing response of an actual aircraft is examined. Specifically, simulation of an F-4 aircraft is used to demonstrate the practicality of the hybrid method. A runway profile that is similar to that traversed by an actual F-4 test plane is used in the numerical model. Internal loads for which experimental data and sufficient structural data is available are chosen to be simulated using the hybrid method. The results are compared with standard time-history analysis and test data to demonstrate the usefulness of the hybrid method as a complementary procedure to time-history analysis.

Although the simulation of a taxiing aircraft is of prime interest, the results of this paper are also applicable to the transient response analysis of other surface tracking vehicles with nonlinear suspension systems. Future applications of the hybrid simulation technique may include the modeling of large space structures that will consist of multiple linear substructures with some nonlinear components. It is envisioned that linear substructures will be attached to one another by nonlinear energy absorbing couplers to prevent vibration propagation. Such nonlinear couplers may be needed to absorb energy input that is originated by attitude control adjustments or by docking space vehicles.

## Time-History Analysis

The first stage of the hybrid analysis procedure is the determination of the nonlinear dynamic forces by time-history analysis on the basis of a small number of orthogonal modal coordinates. The equations of dynamic equilibrium for the aircraft structure are written in terms of the modal coordinates $\eta$[1]

$$[M]\{\ddot{\eta}\}+[C]\{\dot{\eta}\} + [K]\{\eta\} = [\phi]^T\{F-F_s\} \qquad (1)$$

where

[M] = generalized mass matrix
[C] = modal damping matrix
[K] = generalized stiffness matrix
[$\phi$] = mode shape matrix
{F} = applied external loading

The nonlinear load-displacement and load-velocity characteristics of suspension struts are taken into account in terms of the additional $F_s$ nonlinear forces that depend upon the displacement and the rate of displacement of the landing gear. Accordingly, the nodal displacements corresponding to the landing gear attachment points must be calculated at each step during time-history integration to determine the $F_s$ strut forces. Nevertheless, it is advantageous to formulate the generalized vibration problem with the unconstrained aircraft structure to make the rigid body modes orthogonal to the flexible modes and render the generalized mass matrix diagonal. With proper scaling of the mode shape vectors, the generalized masses corresponding to the rigid body modes become the aircraft total mass and mass moment of inertia.[1,3]

Including only a small number of flexible modes (5 to 10) has proven satisfactory in all cases for the time-history simulation of the suspension forces using Eq. (1). It has been found that increasing the number of modes will unduly increase computational effort because of the resulting requirements for the refinement of the time-integration step proportional to the increase in the frequency of the highest mode considered as well as because of the increased number of modal coordinates.

In contrast with the success of time-history simulations for the nonlinear strut forces, the aircraft elastic loads are more difficult to determine on the basis of a small number of generalized modal coordinates. It is essential to include the higher frequency modes to represent the more detailed dynamic response of the structure. To simulate specific local structural response it is necessary to have a sufficiently detailed and accurate model of the aircraft structure to obtain the necessary free-vibration data. It is often quite difficult to obtain a sufficiently accurate model of a structure to yield reliable information on the high frequency vibration modes. However, an equally strong reason for not considering the high frequency modes is the necessary refinement in the time domain discretization that makes a time-history analysis computationally impractical. Without the necessary decrease in the time-integration step size the additional modes will make the analysis less reliable. The development of an alternative to the time-history analysis is also quite useful for validating the convergence of time-history analysis to the correct solution with numerical refinement. The validatory use

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is one reason for the selection of Frequency Domain Analysis as the linear analysis component in the present hybrid formulation. In time-history analysis numerical approximations occur because of discretization in the time domain. In Frequency Domain Analysis discretization is made in the frequency domain. As a result discretization errors occur in opposite directions for the two methods. Thus, if both methods converge to the same solution then convergence to the true solution is verified. Alternatively, the two methods can be used at coarser discretizations to obtain upper and lower bounds to the true solution.

## Linear Analysis in the Frequency Domain

After the time-history determination of the nonlinear suspension strut forces on the basis of a small number of modal coordinates, the detailed analysis of the elastic aircraft structure is carried out in the frequency domain. A greater number of modes can be considered at this stage without concern for the time domain discretization refinement.

The basic conditions that permit a frequency domain analysis are that the nonlinear suspension strut forces are determined and the linear aircraft structure is represented by orthogonal generalized coordinates. At the beginning of the linear analysis, the previously determined nonlinear suspension forces are converted to the frequency domain by Discrete Fourier Transformation (DFT). The DFT coefficients are defined as[1,2]

$$C_{n+1}(\bar{\omega}_n) = \Delta t \sum_{\tau=0}^{N-1} F(\tau) e^{-2\pi i n\tau/N}; n=0,\ldots,N-1 \quad (2)$$

where

$$
\begin{aligned}
i &= \sqrt{-1} \\
\Delta t &= T/N \\
T &= \text{total time period con-} \\
&\quad \text{sidered (includes an} \\
&\quad \text{attached period of } F(t)=0 \\
&\quad \text{to take into account the} \\
&\quad \text{periodic nature of DFT)} \\
\tau &= \text{time step number} \\
n &= \text{frequency step number} \\
N &= \text{number of discrete time} \\
&\quad \text{intervals in } T \\
\bar{\omega}_n &= \text{forcing frequency} \\
C_{n+1}(\bar{\omega}_n) &= \text{complex coefficients that} \\
&\quad \text{define the discretized} \\
&\quad \text{harmonic amplitude} \\
&\quad \text{function in the frequency} \\
&\quad \text{domain}
\end{aligned}
$$

The complex frequency response function, $H_j(\bar{\omega}_n)$, for each $j$th generalized structural mode under the forcing frequency $\omega_n$ is defined as[2]

$$H_j(\bar{\omega}_n) = \frac{1}{-\bar{\omega}^2 M_j + i\bar{\omega}_n C_j + K_j} \quad (3)$$

where

$$
M_j, C_j, K_j = \begin{array}{l} j\text{th generalized modal} \\ \text{mass, damping, and} \\ \text{stiffness, respectively} \end{array}
$$

It can be shown that the total response of a system to any forcing input can be written by means of inverse Fourier transformation. The displacements of the $j$th modal coordinate are given by[2]

$$\eta_j(\tau) = \frac{\Delta\bar{\omega}}{2\pi} \sum_{n=0}^{N-1} H_j(n+1) C_j(n+1) e^{2\pi i n\tau/N} \quad (4)$$

where

$$\Delta\bar{\omega} = \frac{\bar{\omega}_N}{N} = \begin{array}{l} \text{frequency domain dis-} \\ \text{cretization step size} \end{array}$$

Other response parameters such as stresses or loads developed in the structure can be evaluated directly from the generalized modal coordinate displacements. It can be shown that the elastic forces developed due to inertial effects during the dynamic response can be written as[2]

$$\{f(\tau)\} = [m][\phi]\{\omega_j^2 \eta_j(\tau)\} \quad (5)$$

where

$$
\begin{aligned}
\{f(\tau)\} &= \text{dynamic forces at} \\
&\quad \text{structure coordinates} \\
[m] &= \text{structure basic mass} \\
&\quad \text{matrix} \\
[\phi] &= \text{mode shape matrix} \\
\{\omega_j^2 \eta_j(\tau)\} &= \text{a vector of modal ampli-} \\
&\quad \text{tudes, each multiplied by} \\
&\quad \text{the square of the corre-} \\
&\quad \text{sponding modal frequency}
\end{aligned}
$$

Once the dynamic loads $\{f(\tau)\}$ are determined, any internal loading can be computed by passing a section through the selected segment of the structure and writing, at each time step $\tau$, equations of equilibrium that include the inertial forces $\{f(\tau)\}$ as well as other variable and steady state external forces acting on the structure.

## Response of Taxiing Aircraft

The application of the developed hybrid dynamic analysis procedure to taxiing aircraft is the primary subject of this paper. Currently the Air Force is in the process of assessing the operability requirements for specific aircraft on rapidly repaired runways.[4] It is necessary to have an analysis capability that will determine the total aircraft response to possible runway irregularity profiles. With the use of a reliable mathematical model, parametric studies can be made to map out acceptable runway repair patterns and tolerances.

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To determine the response of taxiing aircraft, the equations of dynamic equilibrium are written for the aircraft vehicle and the unsprung masses of the suspension systems. First, the nonlinear time-history analysis is carried out in terms of the rigid-body and flexible modal coordinates. Runway irregularity induced forces are transmitted through tires that are represented by vertical springs. Landing gear unsprung masses transmit the tire loads to the nonlinear suspension struts. The suspension struts are multiple level energy absorption and dissipation systems. Strut forces are highly nonlinear functions of strut deformation and velocity of deformation. These forces are transmitted to the aircraft vehicle at the gear attachment points.

In simulating taxiing aircraft over irregular runways, the bounce and the pitch rigid body modes are significant for the vehicle. The roll mode is not needed due to the symmetry of runway irregularities. Only the vertical modes are considered for the unsprung suspension masses. Vehicle flexibility is taken into account in terms of the elastic free vibration modes. The flexible vibration modes are determined with respect to the unconstrained vehicle structure, thus making the generalized mass matrix diagonal including the rigid body modes. However, because the nonlinear forces depend upon all coordinate deformations, the equations of motion must be solved simultaneously. The FDL-TAXI computer program is used to perform the time-history integrations. The TAXI program is relatively simple compared to other available general purpose time-integration algorithms. Yet, TAXI has been validated by extensive tests and yields excellent simulations for the nonlinear strut forces. Details of the TAXI program are well documented elsewhere.[5]

For the validation of the hybrid analysis method it is desirable to have a detailed numerical model of an aircraft for which test data is available. The data used in this paper pertains to an F-4 aircraft for which some test data is available and structural data is sufficient only to compute the first ten flexible modes reliably. Thus the maximum number of modes that can be used in the linear analysis is limited to ten. The aircraft is assumed to taxi at a steady speed on a symmetrical runway, traversing a standard 78-ft. AM-2 mat that is 1.5 in. high and includes 4 ft. linear ramps at both ends.

Time-history analysis with TAXI is carried out twice. First, the analysis is performed on the basis of 10 modes; second, on the basis of 5 modes. The time-discretization interval is sufficiently small in both cases to obtain convergence for the nonlinear strut forces. The main gear and the nose gear

nonlinear strut forces and the elastic load in shear and bending at the wing root are computed for comparison with test data and hybrid analysis. Fig. 1 shows a com-
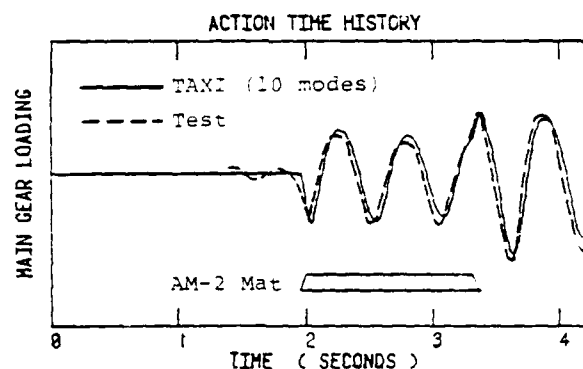


Fig. 1 Comparison of TAXI Simulation with Test

parison of the main gear force time history as determined by TAXI and as measured by testing. In Fig. 1 the AM-2 mat is depicted along the time axis as it is seen from the main gear. The TAXI program is quite successful in predicting the main gear forces very accurately despite some minor differences between the test case and the TAXI model. The simulated model aircraft has slightly different external stores compared to the actually tested plane. Also, the time-history simulation assumes the runway and the AM-2 mat to be perfectly smooth. The TAXI simulation represented by the solid curve in Fig. 1 is carried out on the basis of 10 flexible modes. Fig. 2 shows a comparison of this
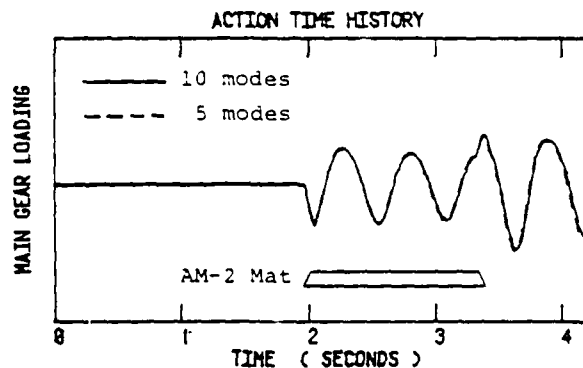


Fig. 2 Comparison of TAXI Simulations on Basis of 5 and 10 Flexible Modes

with a TAXI simulation that is based on only 5 modes. As it is easily seen from Fig. 2 the two simulations give almost

identical results.

Fig. 3 shows a comparison of the nose
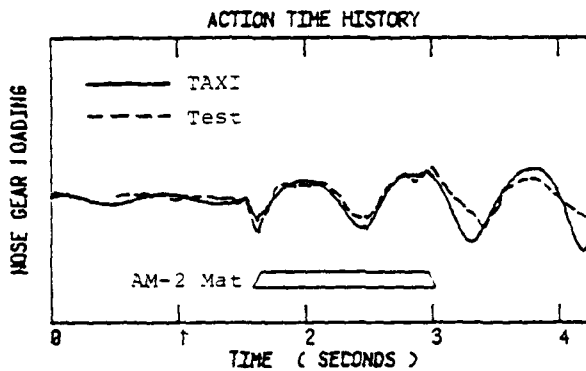
## ACTION TIME HISTORY



Fig. 3 Comparison of TAXI Simulation with Test

gear dynamic loading simulation with experimental data. The solid line in Fig. 3 actually consists of two curves representing TAXI simulations with 5 and 10 modes. These simulations are identical enough to produce the same curve in Fig. 3. The agreement with test data in Fig. 3 is not as good as it was for the main gear loading simulations. This is probably due to the more sensitive nature of the nose gear that may be easily influenced by small irregularities on the actual test runway. However, this difference does not impose a significant impediment for the rest of the analysis as the nose gear load is not very influential in affecting the response characteristics of critical structural components. In Fig. 3 the AM-2 mat is shown as it is observed from the nose gear.

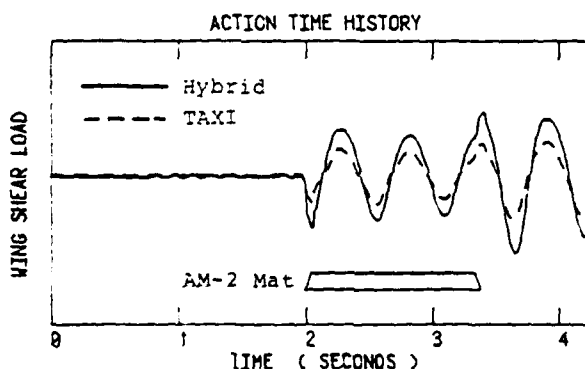Fig. 4 shows a comparison of the wing

## ACTION TIME HISTORY



Fig. 4 Comparison of Hybrid and TAXI Simulations

root shear as determined by TAXI and by the hybrid method. Both curves follow the same pattern but the hybrid analysis produces higher peaks. In examining Fig. 4 it should be recalled that the wing root shear is the algebraic sum of all wing inertial forces acting in the vertical direction plus the lift and gravitational forces acting on the wing and the concentrated load exerted by the main gear that is attached to the wing. The only forces calculated in the hybrid analysis are the inertial components. The main gear forces are determined in TAXI and the gravitational and lift forces are constant at a constant speed. In this particular application the major inertial forces are always opposing the main gear interaction with a very small phase difference. Thus the fundamental contribution to the trend of plots in Fig. 4 comes from the vector difference between the main gear loading and the inertial loads. If only the inertial loads were depicted in Fig. 4 the peaks would be lower in the hybrid method compared to direct evaluation in time-history analysis.

Fig. 5 shows the same curves as in
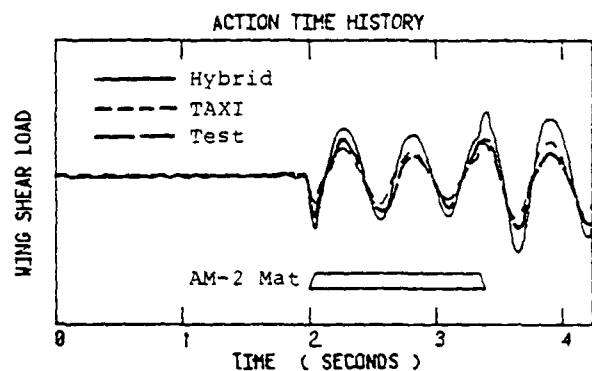
## ACTION TIME HISTORY



Fig. 5 Comparison of Analyses with Test

Fig. 4 with the test results superimposed. It is observed that test results fall between TAXI and hybrid simulations for six out of nine peaks. Using the average value at each extremum location would give a better assessment of the maximum shear load measured by testing.

Fig. 6 shows a comparison of the wing root bending moments computed by direct evaluation in the TAXI program and hybrid analysis. Both methods are based on 10 flexible modes. In this case there is a more significant difference between the two predictions. Fig. 7 is the same as Fig. 6 with the test results superimposed. It is again observed that the test results fall between time-history and hybrid evaluations. It appears that the standard TAXI program can predict the test response
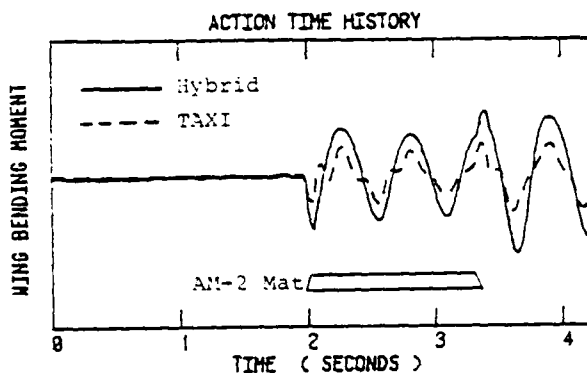
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Fig. 6  Comparison of Hybrid and TAXI
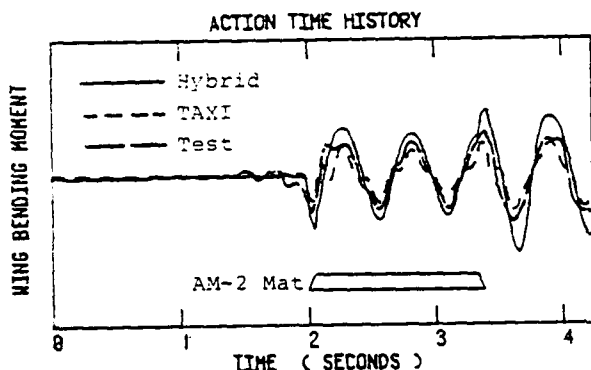Simulations

ACTION TIME HISTORY



Fig. 7  Comparison of Analyses with
Test

pattern better.  However, the peak values
are better predicted if both methods are
used as complementary numerical procedures.

## Conclusion

The utility of the hybrid method has
been demonstrated as a numerical procedure
companion to direct time-history analysis.
It is noted that the hybrid method is a
somewhat novel approach, aimed for the
detailed dynamic analysis of complex
structures with distinct nonlinearities.
Because of limited availability of
structural and test data, it has not been
possible to apply the hybrid method to the
analysis of the type of problems for which
the procedure was formulated.  The lack of
detailed structural data for a tested air-
craft configuration has forced this demon-
stration study to be somewhat rudimentary.
The internal loads simulated in this paper
are heavily influenced by the low frequen-
cy vibration modes of the aircraft

structure.  Accordingly, time-history
analysis is quite satisfactory to evaluate
these internal loads.  The fundamental
appeal in the hybrid formulation is the
capability to evaluate the dynamic re-
sponse that is influenced by the higher
frequency vibration modes, without the
need for excessive refinement of the time-
domain discretization step size.

Despite the inappropriateness of the
simulated dynamic response quantities for
a demonstration case with the hybrid
analysis method, it has been shown that
the hybrid method can be used along with
time-history analysis to obtain a measure
of convergence of the solution to the true
dynamic response.  From a different per-
spective, it can be stated that the time-
history and hybrid analysis methods can be
used to estimate upper and lower bounds to
the peak values of the dynamic response.
This paper represents a first attempt to
evaluate the hybrid method in comparison
with test data.  It is expected that as
the method is further used refinements will
be made to improve the accuracy and the
reliability of the procedure.

The next step in the development of
the hybrid method will be its application
to a tested aircraft for which more
accurate structural information is availa-
ble.  More detailed structural definition
of the aircraft will allow for more
realistic comparison of the simulated re-
sponse with test data.  Also, with the
availability of a more accurate model,
convergence studies in both time and
frequency domain discretization will
become more meaningful.

Finally, the most essential and
appropriate step in the development of the
hybrid method will be the parallel testing
of a model structure in a controlled
loading environment.  In a laboratory
setting complete assurance can be ascer-
tained with regard to the definition of
structural properties, accuracy of the
applied dynamic loading, and the measure-
ment of the dynamic response.  Only with
such reliable data it will be possible to
make the necessary evaluations and re-
finements to render the hybrid method a
truly reliable numerical procedure.

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